Worksheet 3.2 page 1

$l := 1$		length of assembly, m
$X1 := 0$	$X2 := 4\cdot 10^{-3}$	co-ordinates of centre of composite conductors 1 and 2, m
$Y1 := 0$	$Y2 := 0$	
$R1 := 1\cdot 10^{-3}$	$R2 := 1\cdot 10^{-3}$	radius of composite conductors 1 and 2, m.
$n_1 := 12$	$n_2 := 12$	number of elemental conductors in composite 1 and 2
$N := n_1 + n_2$		total number of elemental conductors

defining elemental conductors for composite 1:

$i := 1 .. n_1$ control variable

$$\theta_i := \frac{2\cdot\pi}{n_1}\cdot(i - 0.5)$$

$$x_i := X1 + R1\cdot\cos(\theta_i) \qquad y_i := Y1 + R1\cdot\sin(\theta_i) \qquad r_i := \frac{R1}{n_1}$$

defining elemental conductors for composite 2:-

$i := n_1 + 1 .. N$ extending the control variable

$$\theta_i := \frac{2\cdot\pi}{n_2}\cdot(i - n_1 - 0.5)$$

$$x_i := X2 + R2\cdot\cos(\theta_i) \qquad y_i := Y2 + R2\cdot\sin(\theta_i) \qquad r_i := \frac{R2}{n_2}$$

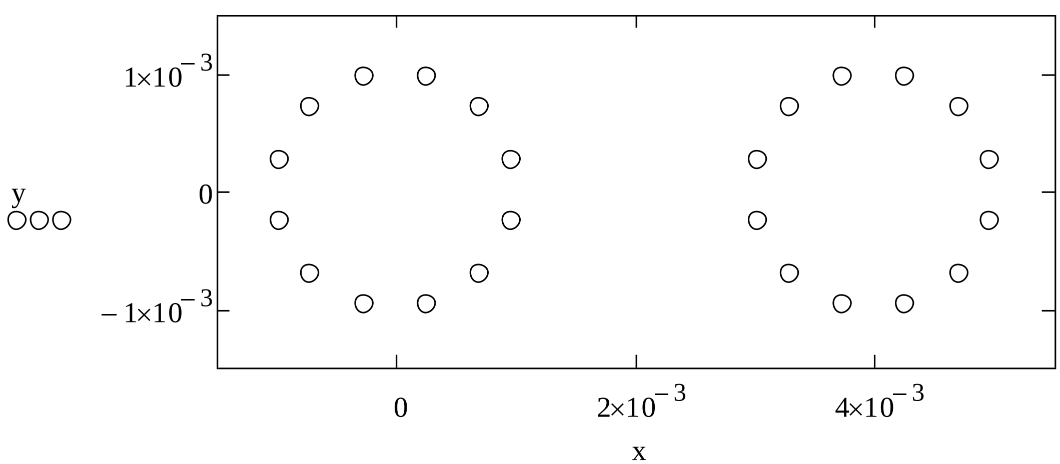

Fig 3.2.5 Deriving the co-ordinates of the elemental conductors

$\mu_o := 4\cdot\pi\cdot 10^{-7}$ H/m $\qquad \mu_r := 1 \qquad K := \dfrac{\mu_o\cdot\mu_r\cdot l}{2\cdot\pi} = 2\times 10^{-7}$ H

$$Zp := \left|\begin{array}{l} \text{for } i \in 1..N \\ \quad \text{for } j \in 1..N \\ \quad\quad \left|\begin{array}{l} h \leftarrow x_j - x_i \\ v \leftarrow y_j - y_i \\ rad \leftarrow \sqrt{h^2 + v^2} \\ rad \leftarrow r_i \ \text{ if } \ rad = 0 \\ Lp_{i,j} \leftarrow K\cdot\ln\left(\dfrac{l}{rad}\right) \end{array}\right. \\ Lp \end{array}\right.$$

See figure 3.1.4

$$Zloop := \left|\begin{array}{l} \text{for } i \in 1..N-1 \\ \quad \text{for } j \in 1..N-1 \\ \quad\quad Lloop_{i,j} \leftarrow Zp_{i,j} - Zp_{i,j+1} - Zp_{i+1,j} + Zp_{i+1,j+1} \\ Lloop \end{array}\right.$$

equation (3.2.3)

$$Vloop := \left|\begin{array}{l} \text{for } i \in 1..N-1 \\ \quad \left|\begin{array}{l} V_i \leftarrow 0 \\ V_i \leftarrow 1 \ \text{ if } \ i = n_1 \end{array}\right. \\ V \end{array}\right.$$

equation (3.2.5)

$$Iloop := \text{lsolve}(Zloop, Vloop)$$

equation (3.2.6)

$$Ip := \left|\begin{array}{l} I_1 \leftarrow Iloop_1 \\ \text{for } i \in 2..N-1 \\ \quad I_i \leftarrow Iloop_i - Iloop_{i-1} \\ I_N \leftarrow -Iloop_{N-1} \\ I \end{array}\right.$$

equation (3.2.7)

Fig 3.2.6 Calculating the currents in the elemental conductors

$\text{Start} := \begin{pmatrix} 1 \\ n_1 + 1 \end{pmatrix}$ $\quad$ $\text{End} := \begin{pmatrix} n_1 \\ n_1 + n_2 \end{pmatrix}$ $\quad$ See figure 3.2.3

$h := 1..2 \quad k := 1..2$ $\quad$ Pointers to sub-matrices

$$vq_{h,k} := \left| \begin{array}{l} v \leftarrow 0 \\ \text{for } i \in \text{Start}_h .. \text{End}_h \\ \quad \text{for } j \in \text{Start}_k .. \text{End}_k \\ \quad\quad v \leftarrow v + Zp_{i,j} \cdot Ip_j \\ \dfrac{v}{n_h} \end{array} \right.$$

equations (3.2.8) to (3.2.12)

$$vq = \begin{pmatrix} 2.622 & -2.122 \\ 2.122 & -2.622 \end{pmatrix} \quad \text{V}$$

$$Vq_h := \left| \begin{array}{l} v \leftarrow 0 \\ \text{for } k \in 1..2 \\ \quad v \leftarrow v + vq_{h,k} \\ v \end{array} \right.$$

check:- voltages along conductors

$$Vq = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix} \quad \text{V}$$

$$Iq_h := \left| \begin{array}{l} I \leftarrow 0 \\ \text{for } i \in \text{Start}_h .. \text{End}_h \\ \quad I \leftarrow I + Ip_i \\ I \end{array} \right.$$

equations (3.2.13) and (3.2.14)

$$Iq = \begin{pmatrix} 1.898 \times 10^{6} \\ -1.898 \times 10^{6} \end{pmatrix} \quad \text{A}$$

$$Lq_{h,k} := \frac{vq_{h,k}}{Iq_k}$$ equation (3.2.15)

$$Lq = \begin{pmatrix} 1.382 \times 10^{-6} & 1.118 \times 10^{-6} \\ 1.118 \times 10^{-6} & 1.382 \times 10^{-6} \end{pmatrix}$$

$$Lc_1 := Lq_{1,1} - Lq_{1,2}$$

$$Lc_2 := Lq_{2,2} - Lq_{2,1}$$ equation (2.5.4)

$$\frac{Lc}{2} = \begin{pmatrix} 1.317 \times 10^{-7} \\ 1.317 \times 10^{-7} \end{pmatrix} \quad \text{H}$$

$$\varepsilon_o := 8.854 \cdot 10^{-12} \qquad \varepsilon_r := 1$$

$$Cc := \frac{\mu_o \cdot \mu_r \cdot \varepsilon_o \cdot \varepsilon_r \cdot l^2}{Lc}$$ equation (2.3.3)

$$Cc = \begin{pmatrix} 4.224 \times 10^{-11} \\ 4.224 \times 10^{-11} \end{pmatrix} \quad \text{F}$$

Figure 3.2.7 Calculating circuit components for composite conductors

$i := 1..N$ $\qquad A_{i,1} := x_i \cdot 10^3$ $\qquad A_{i,2} := y_i \cdot 10^3$ $\qquad A_{i,3} := |Ip_i| \cdot 10^{-3}$

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$$Cloop := \frac{Cc_1 \cdot Cc_2}{Cc_1 + Cc_2}$$

$$Cloop = 2.112 \times 10^{-11}$$

$$b := \frac{X2}{2} \qquad r := R1$$

Electromagnetic Concepts and Applications
GG Skitek & SV Marshall. Pages 208 to 209.
Capacitance between two cylindrical conductors

$$Ctheory := \frac{\pi \cdot \varepsilon_o \cdot \varepsilon_r \cdot l}{\ln\left(\frac{b + \sqrt{b^2 - r^2}}{r}\right)}$$

$$Ctheory = 2.112 \times 10^{-11}$$

Figure 3.2.9 Comparison of results with textbook theory